

Low Energy Pion-Nucleus Interactions: Nuclear Deep Inelastic Scattering, Drell-Yan and Missing Pions

Gerald A. Miller^{a,b}

Department of Physics^a 351560 and Institute For Nuclear Theory^b 351550, University of Washington, Seattle, Washington 98195, USA

Abstract

The experimental discovery that the nucleus is approximately transparent to low energy pions is reviewed. The consequences of this for nuclear deep inelastic scattering and Drell-Yan interactions are discussed. I argue that low energy nucleus data imply that there is little nuclear enhancement of the pion cloud of a nucleon, and try to interpret this in terms of nucleon-nucleon correlations.

Text of invited talk presented at the LAMPF Users Group, Inc. Symposium, Oct. 25-26, 1996

DOE/ER/41014-3-N97

DOE/ER/40561-299-INT96-19-06

I. INTRODUCTION

I will discuss low energy pion-nucleus interactions and the relationship between that subject and nuclear deep inelastic and Drell-Yan interactions and the problem of the missing pions.

I begin by mentioning my enthusiastic appreciation of two very successful parts of the Los Alamos Meson Physics Facility LAMPF program. The first is the inelastic reactions in the Δ resonance region. The π^+ interact mainly with protons and the π^- with neutrons. This feature was used to separate the proton and neutron contributions to inelastic transition densities at the nuclear surface. Many times the independently obtained proton contributions were consistent with those derived from electron scattering, thereby lending credence to the neutron results. One exciting spin-off was the ability to observe isospin mixing.

I also want to record my enthusiasm for the double charge exchange (π^+, π^-) program. The experimentalists observed nucleon-nucleon correlations. These were not of the six quark bag type that I was excited about, but were correlations nonetheless!

Now I come to the outline of my own discourse, which concerns low energy (≈ 50 MeV) pion-nucleus interactions. LAMPF and TRIUMF physicists discovered that the nucleus is transparent – pions have essentially no multiple scattering in the nucleus. I discuss some of the evidence obtained from four different kinds of measurements. It turned out that related studies of the π -nucleus interaction were being made at CERN in (μ, μ') nuclear deep inelastic scattering. Nuclear effects on measured nucleon structure functions were observed – the celebrated EMC effect. Models in which the nucleus enhanced the pion cloud of a nucleon, pion enhancement models (in which the pion bounced off many nucleons) were successful in reproducing the data. There were many other models which were able to reproduce the early data. Our response was to suggest that the nuclear Drell-Yan process $pA \rightarrow \mu^+ \mu^- + X$ could be used to test such models - Los Alamos physicists responded by carrying out experiment 772 at FNLAB. One of the results is that there is no evidence for a pion excess. This is the famous missing pions problem. Perhaps this result should not have

been surprising, in view of the low energy pion scattering measurements made at LAMPF and TRIUMF. I will then discuss a possible resolution of this problem.

II. 50 MEV π^- INTERACTION

The average π nucleon cross section is about 25 mb at this energy. But the nuclear scattering requires sufficient momentum to push the bound nucleon above the Fermi sea. Including this feature leads [1] to an effective cross section of 15 mb, and a fairly long mean free path of 4 fm. Thus a pion travels a long distance as it moves through the nucleus.

The idea that low energy pions do not interact much is the community view. Here is a quotation from a well known text book [2]:

“In view of the relative weakness of the low energy interaction, the qualitative features of the π -nucleus scattering show up already in the first-order Born approximation.”

An example is the minimum caused by s-p interference in the elastic scattering angular distribution which has a position at an angle independent of nuclear target. This minimum is reproduced by using the first-order Born approximation.

The relative weakness should have simplified various analyses. But the p-wave nature of the π N interaction caused big (erroneously computed) multiple scattering effects.

III. P-WAVE INTERACTION

The P wave part of the average π N scattering amplitude $f(\vec{k}, \vec{k}')$ can be represented as

$$4\pi f(\vec{k}, \vec{k}') = \tilde{b}_1 \vec{k} \cdot \vec{k}' \quad (1)$$

where \vec{k} and \vec{k}' are the final and initial relative pion-nucleon momenta. The optical potential U is given the product of operators

$$U = 4\pi f \rho \quad , \quad (2)$$

which becomes the Kisslinger potential

$$U(r) = b_1 \vec{\nabla} \cdot \rho(r) \vec{\nabla} , \quad (3)$$

when evaluated in coordinate space.

Consider the situation of the nuclear center where ρ is approximately constant. Then the left-most gradient in Eq. (3) can be pulled through to the right and the Klein Gordon equation reads

$$-\nabla^2 + b_1 \rho \nabla^2 \psi = k^2 \psi \quad (4)$$

where k is the on-shell wave number. One may find a solution of the form

$$\psi(\vec{r}) = e^{i\vec{K} \cdot \vec{r}} \quad (5)$$

with

$$K^2 = \frac{k^2}{1 - b_1 \rho} . \quad (6)$$

The above result does not look dramatic until one realizes that

$$b_1 \rho \approx 1 \quad (7)$$

so that K^2 is very large. This spurious effect can have bad calculational consequences. This is known as the Kisslinger singularity, but I want to stress that Leonard Kisslinger did not encounter this problem in deriving his potential – he worked wisely and correctly in the Born approximation.

Theorists were confronted by the problem of removing the singularity.

IV. FIXES OF THE KISSLINGER SINGULARITY

Two different methods were used. The first was to modify the scattering amplitude or t-matrix for off-shell momenta:

$$4\pi f(\vec{k}, \vec{k}') = v(k)\vec{k} \cdot \vec{k}' v(k') \quad (8)$$

with

$$\lim_{k \rightarrow \infty} v(k) = 0 \quad , \quad (9)$$

as suggested by the finite size of the nucleon. The high momentum amplitude is cut off, and multiple scattering is suppressed as a result. There were many such models [3-6] of this kind, and the desire to obtain a quark-based πN interaction was a prime motivation for the cloudy bag model[7].

The second method invoked the repulsive nucleon-nucleon correlations—this was the EELL potential [8]. I will discuss this according to the argument presented in Ref. 9.

Consider the second order scattering operator $\hat{T}^{(2)}$ from two nucleons separated by \vec{r} :

$$\hat{T}^{(2)} \sim \int \frac{d^3k}{(2\pi)^3} e^{i\vec{K} \cdot \vec{r}} \frac{[(\vec{k} \cdot \vec{K} \vec{K} \cdot \vec{k} - \frac{1}{3}k^2 K^2) + \frac{1}{3}k^2 K^2]}{k^2 - K^2 + i\epsilon} \quad (10)$$

where virtual momentum K is the integration variable and k is the on-shell wave number. The factor $\vec{k} \cdot \vec{K} \vec{K} \cdot \vec{k}$ arises from two p-wave interaction vertices. A term $\frac{1}{3}k^2 K^2$ has been subtracted from and added to the numerator to provide a decomposition into invariants. The two terms in the numerator can be shown to be very small. Integration over d^3K leads to

$$\frac{1}{3}k^2 K^2 \rightarrow \frac{k^2}{3} [-\delta(\vec{r}) - k^2 \frac{e^{ikr}}{4\pi r}] \quad , \quad (11)$$

with the first term vanishing because of the correlations and the second of order k^4 and therefore small. The remaining term is a tensor. The integration over \vec{K} gives

$$\vec{k} \cdot \vec{K} \vec{K} \cdot \vec{k} \rightarrow (\vec{k} \cdot \hat{r} k \cdot \hat{r} - \frac{1}{3}k^2) \quad (12)$$

which after integration over \vec{r} becomes

$$(\vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2 - \frac{1}{3}k^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2) \quad , \quad (13)$$

which averages to zero. So there is very little second order or, by implication, higher order scattering.

Both methods lead to the result that there is little multiple scattering. It is interesting to recall that there were big fights over which method is better. These days, everyone knows that Lagrangians can take on a variety of forms, generated by making field redefinitions. It is very likely that one can interpolate between the two methods with such a change of variables.

There is a physics question remaining. Although the multiple scattering is small—how small is it? Here is where the experimentalists came to the forefront by supplying beautifully relevant data.

V. SINGLE CHARGE EXCHANGE ON PROTON AND NUCLEAR TARGETS

The reaction $\pi^- p \rightarrow \pi^0 n$ or $\pi^+ n \rightarrow \pi^0 p$, equal by charge symmetry, was studied with the wonderful π^0 spectrometer. Here

$$\frac{d\sigma}{d\Omega}(\pi^- p \rightarrow \pi^0 n) = \frac{d\sigma}{d\Omega}(\pi^+ n \rightarrow \pi^0 p) = |A + B\vec{k} \cdot \vec{k}'|^2 \quad (14)$$

where A represents the repulsive Weinberg-Tomazowa S-wave term and B the attractive p-wave term. There is a tendency to cancel for forward scattering angles with $\vec{k} \cdot \vec{k}' = k^2$ which becomes exact near 50 MeV. This cancellation is shown by the data displayed in Fig. 3 of Ref.10. The zero in the forward scattering amplitude affords the opportunity to learn about multiple scattering in the initial π^+ and final π^0 state interactions. If such replaces k^2 by K^2 with $K^2 \gg k^2$, the cancellation will not occur. The very same Fig. 3 of Ref. 10 shows that the zero is maintained at approximately the same energy for nuclear targets throughout the periodic table. The simplest conclusion is that any multiple scattering must be very weak.

VI. LOW ENERGY ($\pi_1\pi'$) INTERACTIONS

The reaction $\pi + A(O_1^+) \rightarrow \pi^+ + A^+(O_2^+)$ also affords a study of multiple scattering. In Born approximation the scattering amplitude is the Fourier transform of the product of the initial and final wave functions. If the momentum transfer vanishes, orthogonality causes a vanishing scattering amplitude and a backward peaked cross section. (The momentum transfer is not exactly zero so that one does not expect an exact zero in the forward direction.) Strong multiple scattering leads to a forward peak as seen in (α, α') reactions. The ^{12}C measurements [11,12], especially Fig. 2 of Ref. 11 indeed show such a backward peaking, which is in agreement with an EELL calculation of Ref[11]. The angular distribution obtained from a Kisslinger potential which exactly reproduces the elastic data has a strong forward peak (see Fig. 11 of Ref.12) which disagrees with the inelastic data by about an order of magnitude or more. The backward peaked angular distribution provides another signature for the approximate transparency of nuclei to pions.

VII. π -NUCLEUS TOTAL CROSS SECTIONS

Measurements of total cross sections provide a test of transparency or lack of multiple scattering. If only single scattering occurs, the total cross section will be proportional to the number of nucleons, A , in a nucleus. The experiments of Saunders, et al [13] indeed found such a relation:

$$\frac{\sigma_{tot}(\pi A)}{A} \approx \sigma(\pi N) \quad . \quad (15)$$

The results are plotted in Fig. 8 of Ref.13. The π^+ data satisfy Eq. (15) very precisely. The π^- data show a slight rise with A which is presumably due to higher order effects of the attractive π^- - nuclear Coulomb interaction.

I have discussed evidence from elastic angular distributions, single charge exchange reactions, inelastic scattering and total cross sections. These indicate that the nucleus is approximately transparent to pions. This was discovered during the 1970's, 1980's and 1990's.

It is interesting to realize that other investigations of the nuclear pion content were being carried out at CERN.

VIII. EMC EFFECT

The European Muon Collaboration (EMC) was involved with deep inelastic μ scattering experiments to determine the $F_2(x)$ structure function of nucleons. Experiments were done on Fe nuclei to increase the cross section with the expectation that the nucleus looks like A free nucleons in these high $Q^2 \sim 100 \text{ GeV}^2$ reactions. This was a common popular assumption. The data for the ratio of Fe to D structure functions showed otherwise. The structure function of a bound nucleon depends on its nuclear surroundings. The first data [14] showed that the structure function ratio was about 1.15 at $x = 0.05$ and dropped approximately linearly with x until x reached $\simeq 0.65$. See Fig. 2 of Ref. 14 which shows the significant systematic errors. This EMC effect was astounding. Nuclei affect quark distributions. Quarks were once and forever part of nuclear physics.

IX. ERICSON-THOMAS[15] PION ENHANCEMENT MODEL

It was natural for nuclear theorists to attempt to explain the EMC effect using nuclear mechanisms. I think the best such attempt was the work of Ericson and Thomas. Pions contribute to deep inelastic scattering from a nucleon because a nucleon can emit a virtual pion. The incident γ^* then can bash this pion into bits. This process can be enhanced in a nucleus because the virtual pion can multiple scatter amongst all the target nucleons. See Fig.1 of Ref.15. Intermediate nucleon and Δ -hole states are formed. The repulsive particle-hole interaction is mocked up by the Landau-Migdal parameter g' . The virtual pions of momentum 300-400 MeV/c were strongly enhanced in that calculation, hence the name pion enhancement model. The resulting calculations, shown in Fig. 2 of Ref. 15 reproduced the early EMC data. The data of Refs. 10-13 were not yet available.

X. EMC MEANS EVERYONE'S MODEL IS COOL

There were many models that could reproduce the EMC effect. Examples are shown in Fig.1 of Ref.16. One could use six quark bags with structure functions different from those of free nucleons, pion enhancement models, linear combinations thereof, and dynamical rescaling which invokes the assumption that the scale parameters which governs perturbative QCD evaluation depends on the nucleus. All such provided a qualitative reproduction of the growing set of nuclear DIS data, except in the shadowing region which occurs at low x .

So there were many cool models, and it became necessary to derive tests to see which of these (if any) were really correct. My response was to suggest that Drell Yan $\mu^+\mu^-$ pairs be measured in high energy proton-nucleus interactions.

XI. DRELL YAN DISENTANGLES THE EMC EFFECT [16]

I was serving on a committee to develop a proposal for LAMPFII with a homework assignment of dreaming up new experiments. I thought that the Drell Yan process

$$p + A \rightarrow \mu^+\mu^- + X \tag{16}$$

which occurs, for example, when a quark from the proton beam annihilates with an antiquark from the nuclear target to form a virtual time like virtual photon. The signature is the γ^* decay into easily detectable $\mu^+\mu^-$ pairs.

Our calculations [16] showed that the different models, behaving similarly in deep inelastic scattering, acted very differently with the Drell Yan probe. There was an existing set up for Expt 605 already in place at Fermilab. The Los Alamos physicists were thus able to do Drell Yan measurements [17].

XII. NUCLEAR DEPENDENCE OF DRELL YAN EXPT 772 [17]

This experiment measured cross section ratios: $C/{}^2\text{H}$, $\text{Ca}/{}^2\text{H}$, $\text{Fe}/{}^2\text{H}$, $\text{W}/{}^2\text{H}$ the result that there was no excess of pions or anything else. There was very little A dependence.

See Fig. 3 of Ref. 17. The ratio for Fe/ ^2H was the one most studied theoretically and pion excess and quark cluster model calculations yielded ratios much larger than unity, in serious disagreement with the E772 data. The paper, Ref.17, displayed an incorrect pion excess calculation, but an improved one, Fig. 3 (with $g'=0.7$) of Ref.18, obtained a ratio of 1.1, also in disagreement with the data. The dynamical rescaling model appeared to agree with the Drell Yan data, but when used with the same parameters, disagrees with the deep inelastic scattering data [19].

No pion excess was seen. A similar result was obtained with the (p,n) and (\vec{p}, \vec{n}) reactions. See Rappaport's paper at this conference.

This failure to observe the pions was a quite a puzzle. Bertsch, Frankfurt and Strikman wrote a paper "Where are the nuclear pions," See Ref. 20.

XIII. WHY PIONS ARE NOT ENHANCED IN NUCLEI

I shall present two arguments. The first is that we should have expected no enhancement, after 1985 or so, because the low energy pion-nucleus data showed that not much multiple scattering occurs. It is this very same multiple scattering which provided the pion enhancement in Ref.15. It is true that the pions in deep inelastic scattering are virtual and space like. But this should diminish the interactions since these pions are further away from the nucleon pole and Δ resonance. Thus we now know that the low energy pion-nucleus data imply that there should be little, if any, pion enhancement.

It would also be satisfying to understand this using the theory. Mark Strikman (PSU) and I are currently working on this problem. We wanted to start with some simple, understandable calculation. We started by computing the pion excess per nucleon δn_π in the deuteron. There are two contributions. The first is due to the change in the single nucleon term due to the binding. The energy denominator of the pion-two-nucleon intermediate state is larger than for the corresponding π -nucleon state, because of the extra kinetic energy of the spectator nucleon. The resulting contribution to the δn_π is small and negative. This is

a new term.

There is another term due to the pion exchanged between two nucleons; this gives a small positive contribution which is cancelled approximately by the new term mentioned above. The exchange term is small because the deuteron wave function does not support the exchange of a pion of momentum of about 400 MeV/c.

So there is almost no pion excess in the deuteron in our evaluation. One might argue that this would not occur for heavy nuclei. Indeed there is no $N\Delta$ intermediate state in the deuteron, while such are allowed in heavy nuclei. However, the virtual Δ can not propagate a great distance and correlations should suppress their effects. Furthermore, the short range correlations in nuclei are very similar to those of the deuteron wave function, and we expect to get a small pion excess in heavy nuclei.

We further speculate that any model calculation producing a large pion excess will also produce low energy pion nucleus calculations in disagreement with the low energy data.

I would like to discuss the fundamental difference between our deuteron multiple scattering calculations and those of Ref.15. One can compare diagrams of the same order to get an idea. In Ref.15 the repulsion is represented by a constant g' . The deuteron matrix elements contain factors of $\int d^3p \psi^\dagger(\vec{p} - \vec{q}/2) \cdots \psi(\vec{p} - \vec{q}/2) \propto F(q)$. When two nucleons exchange a pion of momentum \vec{q} , this nuclear form factor cuts off the contributions for high \vec{q} . My belief is that no single parameter g' , no matter how well motivated, can do the job of a whole function. Indeed, this is one of the main mechanisms in another approach [20] towards understanding why the pions are not enhanced.

XIV. SUMMARY

I list the major points.

- The (π, π') and (π^+, π^+, π^-) programs were major successes
- Low energy π reactions show that the nucleus is transparent to pions
- This transparency is consistent with nuclear Drell Yan data

- Early π enhancement predictions were based on good ideas, but an oversimplified theory.

There is still a problem. To my knowledge there is no theoretically viable model which explains both the nuclear deep inelastic and Drell Yan data.

I thank the national Institute for Nuclear Theory for its support. This work is supported in part by the USDOE under Grants DE-FG03-97ER41014 and DE-FG06-88ER40427.

REFERENCES

- [1] R. Landau *et al.*, *Phys. Rep.* **58**, 121 (1980).
- [2] T.E.O. Ericson *et al.*, *Phys. Rep.*”Pions and Nuclei (1988), Oxford Univ. Press, New York
- [3] R. Landau *et al.*, *Ann. Phys. (NY)* **78**, 299 (1973).
- [4] D.J. Ernst *et al.*, *Phys. Rev. C* **10**, 1708 (1974).
- [5] J.T. Londergan *et al.*, *Ann. Phys. (NY)* **86**, 147 (1974).
- [6] M.G. Piepho *et al.*, *Phys. Rev. C* **9**, 1352 (1974).
- [7] S. Th  berge, *et al. Phys. Rev. D* **22** (1980) 2838; **D23** (1981) 2106(E). A. W. Thomas, *et al, Phys. Rev. D* **24**, 216 (1981); A. W. Thomas, *Adv. Nucl. Phys.* **13** (1984) 1; G. A. Miller, *Int. Rev. Nucl. Phys.* **2**, 190 (1984).
- [8] M. Ericson *et al.*, *Ann. Phys. (NY)* **36**, 383 (1966).
- [9] G.E. Brown *et al.*, *Phys. Rep.* **50**, 227 (1979).
- [10] F. Irom *et al.*, *Phys. Rev. Lett* **55**, 1862 (1985).
- [11] L. Lee *et al.*, *Phys. Lett. B* **174**, 147, (1986).
- [12] J.F. Amann *et al.*, *Phys. Rev C* **23**, 1635, (1981).
- [13] A. Saunders *et al.*, *Phys. Rev. C* **53**, 1745, (1996).
- [14] J. Aubert *et al.*, *Phys. Lett B* **123**, 275 (1982).
- [15] M. Ericson *et al.*, *Phys. Lett. B* **128**, 1121 (1983).
- [16] R.P. Bickerstaff *et al.*, *Phys. Rev. Lett.* **53**, 2532 (1984).
- [17] D.M. Alde *et al.*, *Phys. Rev. Lett.* **64**, 2479 (1990).
- [18] H. Jung *et al.*, *Phys. Rev. C* **41**, 659 (1990).

- [19] H. Jung, private communication to G.A. Miller.
- [20] G.F. Bertsch *et al.*, *Science* **259**, 773 (1993).
- [21] G.A. Brown *et al.*, *Nucl. Phys. A* **593**, 295 (1995).